

Spinning super-massive objects in galactic nuclei up to $a_* > 1$

Cosimo Bambi*

*Institute for the Physics and Mathematics of the Universe,
The University of Tokyo, Kashiwa, Chiba 277-8583, Japan*

(Dated: May 24, 2011)

Nowadays we believe that a typical galaxy contains about 10^7 stellar-mass black holes and a single super-massive black hole at its center. According to general relativity, these objects are characterized solely by their mass M and by their spin parameter a_* . A fundamental limit for a black hole in general relativity is the Kerr bound $|a_*| \leq 1$, but the accretion process can spin it up to $a_* \approx 0.998$. If a compact object is not a black hole, the Kerr bound does not hold and in this letter I provide some evidences suggesting that the accretion process could spin the body up to $a_* > 1$. While this fact should be negligible for stellar-mass objects, some of the super-massive objects at the center of galaxies may actually be super-spinning bodies exceeding the Kerr bound. Such a possibility can be tested by gravitational wave detectors like LISA or by sub-millimeter very long baseline interferometry facilities.

PACS numbers: 04.50.Kd, 04.70.Bw, 97.60.Lf

Introduction — There are robust observational evidences of the existence of 5 – 20 Solar mass compact bodies in X-ray binary systems [1] and of $10^5 - 10^9$ Solar mass bodies at the center of most galaxies [2]. All these objects are commonly interpreted as black holes (BHs) because they cannot be explained otherwise without introducing new physics. In particular, stellar-mass objects in X-ray binary systems are too heavy to be neutron or quark stars for any reasonable matter equation of state [3]. Observations of stellar orbits around the super-massive BH candidate at the center of the Galaxy show that this object is too massive, compact, and old to be a cluster of non-luminous bodies [4].

In 4-dimensional general relativity, BHs are known as Kerr BHs and are completely specified by their mass, M , and by their dimensionless spin parameter, $a_* = J/M^2$, where J is the BH spin angular momentum. The mass M sets the size of the system and, in principle, it can assume any value. The spin parameter a_* determines the properties of the space-time. It must satisfy the constraint $|a_*| \leq 1$, which is the condition for the existence of the event horizon.

In general, the value of the spin parameter is determined by the competition of three physical processes: the event creating the object, mergers, and gas accretion. Recent simulations taking the relevant microphysics processes into account find that the BH formed after the collapse of a super-massive star has $a_* \sim 0.5 - 0.8$, and then it is spun up to $a_* \sim 0.6 - 0.9$, depending on the initial angular velocity of the collapsing stellar core [5]. In the case of the merger of two neutron stars, the product of the coalescence is a BH with $a_* \approx 0.78$, depending only weakly on the total mass and mass ratio of the system [6]. The capture of small bodies (minor mergers) in randomly oriented orbits typically spins a BH down [7, 8], because the magnitude of the orbital angular momentum for corotating orbits is smaller than the one for counterrotating orbits. The case of coalescence of two BHs with compa-

rable mass (major merger) has been addressed only in the last few years [9]. For random mergers, the most probable final product is a BH with $a_* \approx 0.70$, while fast-rotating objects with $a_* > 0.9$ should be rare [10].

The gas in an accretion disk falls to the BH by losing energy and angular momentum. When it reaches the innermost stable circular orbit (ISCO), it is quickly swallowed by the BH, which changes its mass by $\delta M = \epsilon_{\text{ISCO}} \delta m$ and its spin by $\delta J = \lambda_{\text{ISCO}} \delta m$, where ϵ_{ISCO} and λ_{ISCO} are respectively the specific energy and the specific angular momentum of a test-particle at the ISCO, while δm is the gas rest-mass. The evolution of the spin parameter is thus governed by the following equation [11]

$$\frac{da_*}{d \ln M} = \frac{1}{M} \frac{\lambda_{\text{ISCO}}}{\epsilon_{\text{ISCO}}} - 2a_*. \quad (1)$$

If the disk is on the BH equatorial plane, the equilibrium is reached for $a_*^{eq} = 1$ by swallowing a finite amount of matter. For example, an initially non-rotating BH reaches the equilibrium after increasing its mass by a factor $\sqrt{6} \approx 2.4$ [11]. Including the effect of the radiation emitted by the disk and captured by the BH, one finds $a_*^{eq} \approx 0.998$ [12], because radiation with angular momentum opposite to the BH spin has larger capture cross section. The presence of magnetic fields in the plunging region may further reduce this value to $a_*^{eq} \sim 0.95$ [8, 13], by transporting angular momentum outward.

For stellar-mass BHs, the processes of mergers and gas accretion are more likely negligible. If they belong to low-mass X-ray binary systems, even swallowing the stellar companion they cannot change significantly their spin. If they are in high-mass X-ray binary systems, even accreting at the Eddington limit they do not have enough time to grow before the explosion of the companion. So, the value of the spin of stellar-mass BHs should reflect the one at the time of their creation. On the contrary, for super-massive objects the spin is determined by their evolution history, since they have increased their mass by

a few orders of magnitude from the original value. For prolonged disk accretion, the BH has the time to align itself with the disk, the process of gas accretion dominates over mergers, and the low-redshift super-massive BH population in galactic nuclei would be characterized by fast-rotating objects, $a_* > 0.9$, with some of them possibly close to the theoretical bound $a_* \approx 0.998$ [10, 14].

Since there are no clear evidences that the current BH candidates are really the BH predicted by general relativity, it is worth investigating other possibilities. In this letter I discuss the accretion process from a thin disk in a space-time with deviations from the Kerr metric and I argue that the central object may be spun up to $a_* > 1$. Such a possibility is a new finding, but it does not contradict any rule or belief, because the Kerr bound $|a_*| \leq 1$ is defined only for BHs. This possibility is well known for objects such as main-sequence stars and white dwarfs. However, it does not seem so trivial that this is also the case for extremely compact objects. The key ingredient is that any deviation from the Kerr geometry makes the inner radius of the disk of very fast-rotating objects significantly larger than the one of BHs. In most cases, this leads to a higher value of the quantity $\lambda_{\text{ISCO}}/\epsilon_{\text{ISCO}}$ in Eq. (1) and may move a_*^{eq} beyond 1. The super-massive objects in galactic nuclei may thus be super-spinning objects violating the Kerr bound [15].

Non-Kerr compact objects — The Manko-Novikov (MN) metric is a stationary, axisymmetric, and asymptotically flat exact solution of the vacuum Einstein equations [16]. It is not a BH solution, but it can be used to describe the gravitational field outside a generic body. The expression of the metric is quite long and can be seen in [17], where we corrected a few typos of the original article by Manko and Novikov. The solution has an infinite number of free parameters, which determine the mass, the spin, and all the higher order mass multipole moments of the gravitational field. For the sake of simplicity, here I consider only three parameters, determining the mass M , the spin J , and the quadrupole moment Q . The latter can be parametrized by the anomalous quadrupole moment q , defined by

$$Q = Q_{\text{Kerr}} - qM^3, \quad (2)$$

where $Q_{\text{Kerr}} = -a_*^2 M^3$ is the quadrupole moment of a BH. For $q = 0$ we recover exactly the Kerr metric, while for $q > 0$ ($q < 0$) the object is more oblate (prolate) than a BH. Another interesting parametrization is

$$Q = -(1 + \tilde{q})a_*^2 M^3, \quad (3)$$

presumably with $\tilde{q} \geq -1$, since it is difficult to imagine that the rotation makes the object more and more prolate.

For a disk on the equatorial plane, ϵ_{ISCO} and λ_{ISCO} can be computed numerically as described in [17] and then one can evaluate $da_*/d \ln M$. The result is plotted in Fig. 1. For objects more oblate than a BH, the right

hand side of Eq. (1) is always positive, suggesting that $a_*^{\text{eq}} > 1$. For objects more prolate than a BH, one finds two different cases. When $q \gtrsim -2$, $da_*/d \ln M$ becomes null before $a_* = 1$: in this case $a_*^{\text{eq}} < 1$, with a minimum $a_*^{\text{eq}} \approx 0.92$ at $q \approx -0.3$. When $q \lesssim -2$, $da_*/d \ln M$ is always positive and presumably $a_*^{\text{eq}} > 1$.

Fig. 1 can be interpreted by considering the behavior of the inner radius of the disk, see Fig. 2 and, for more details, Appendix B of Ref. [17]. It turns out that the inner radius of the disk of generic fast-rotating objects is always larger than in the BH case and this is roughly reflected in a higher value of $\lambda_{\text{ISCO}}/\epsilon_{\text{ISCO}}$. For $q > 0$, the inner radius can be found as in the Kerr metric. For $q < 0$, the picture is more complex. For any spin parameter a_* , there are two critical values, say q_1 and q_2 with $q_1 > q_2$. If $q > q_1$, there are no differences with the $q \geq 0$ case. If $q_2 < q < q_1$, there are two disconnected regions with stable circular orbits: the standard region $r > r_1$ and an internal one with $r_3 < r < r_2$ ($r_2 < r_1$). Since the orbits in the internal region have higher energy and angular momentum than the orbit at r_1 , the gas cannot go from r_1 to an orbit in the internal region, and the inner radius of the disk is thus r_1 . As q decreases, r_1 and r_2 approaches each other and, for $q \leq q_2$, we have only one region with internal radius r_3 . At $q = q_2$ we have thus a sudden decreases of the inner radius of the disk, as we can see in Fig. 2. The discontinuities of the curves in Fig. 1 for $q = -0.3$ and $\tilde{q} = -0.3$ and -1.0 correspond to this transition. For $q = -1.0$ and -3.0 , the transition occurs at $a_* < 0$. Let us notice that when a_* approaches 1, q_1 and q_2 approach 0.

Here I have only shown the case of objects whose gravitational field has quadrupole deformations with respect to the Kerr metric. However, the result seems to be much more general. It turns out that only in an exact Kerr metric the radius of the ISCO goes to M and that any deviation from the Kerr background makes the inner radius of the disk significantly larger. While this does not strictly mean that $a_*^{\text{eq}} > 1$ for non-Kerr objects (see the case $-2 \lesssim q < 0$), the space region with $a_*^{\text{eq}} < 1$ is always limited. The reason of this peculiarity of the Kerr metric is presumably related to the fact that only in this special case the space-time is “regular”. For $q \neq 0$, there are some pathological features (e.g. closed time-like curves) at very small radii: they can be neglected in our study, because they are inside the inner radius, and the idea is indeed that this region does not exist in reality, because inside the object, while the MN solution would describe the exterior gravitational field. Since the inner radius of the disk turns out to be always outside the pathological region, except in the case of an exact Kerr metric it can never goes to M as a_* approaches 1.

Discussion — When the right hand side of Eq. (1) is positive, the accreting gas spins the object up. If we admit deviations from the Kerr geometry, $da_*/d \ln M$ can be positive at $a_* = 1$ (while it is zero for a BH) and this

suggests the possibility of an equilibrium spin parameter exceeding the Kerr bound. Unfortunately, the MN solution requires $|a_*| < 1$ and therefore it is impossible to predict a_*^{eq} . However, this is likely only a problem of coordinates. The same problem exists for the Kerr case: the MN solution is in prolate quasi-cylindrical coordinates and requires $|a_*| < 1$, while if we use Boyer-Lindquist or Kerr-Schild coordinates we can consider objects with $|a_*| > 1$, even if they are not BHs but naked singularities.

Since we do not know the nature of these objects, we cannot say which value of q is reasonable. We can notice that for any common equation of state a rotating object is more oblate than a BH (that is, q and $\tilde{q} > 0$) and that for a neutron star one expects a quadrupole moment Q as in Eq. (3), with $\tilde{q} \approx 1 - 10$, depending on the equation of state and the mass of the object [18]. So, it is not unreasonable to expect sizable deviations from the theoretical bound for BHs $a_* \approx 0.998$, even including the (small) effect of capture of radiation emitted by the disk [12] and the (more important) one of magnetic fields in the plunging region [8, 13].

In the case of non-Kerr objects, we can neither predict the natal spin, nor the outcome of major mergers, because we do not know their internal structure. Like for BHs, minor mergers spin the object down, since the magnitude of the angular momentum of particles in counterrotating orbits is larger than the one of particles in corotating orbits. However, for prolonged disk accretion, the other mechanisms are not important as long as the timescale of the alignment of the spin of the object with the disk is much shorter than the time for the mass to increase significantly. For a BH, the alignment timescale is determined by the coupling between its spin and the gas orbital angular momentum and is [19]

$$t_{\text{align}} \sim \frac{a_* M^{3/2}}{\dot{M} R_W^{1/2}} \left(\frac{\nu_1}{\nu_2} \right), \quad (4)$$

where \dot{M} is the mass accretion rate, $R_W \sim 10^4 M$ is the warp radius, while ν_1 and ν_2 are respectively the viscosities acting in the plane of the disk and normal to the disk. For a thin disk, t_{align} can be much shorter than the accretion timescale $t_{\text{acc}} \sim M/\dot{M}$ [10, 20]. The same estimate holds for a generic body as long as $|\tilde{q}| \ll 100$, because t_{align} is still determined by the spin: the angular frequency of the precession induced by the spin is $\omega_J \sim J r^{-3}$, while the one induced by the quadrupole moment is $\omega_Q \sim Q M^{-1/2} r^{-7/2}$.

Till now, I neglected the possibility that these objects become unstable before reaching a_*^{eq} . At least for small deviations from the Kerr metric, one can expect instabilities for a_* a bit larger than 1 if the object is so compact to have an external ergoregion [21]. The object may also become unstable before the spin parameter reaches 1, depending on its equation of state, just like a neutron star can never spin up to a rotational period shorter than

about 1 ms due to r-mode instabilities [22]. Since the process of spin-up by the accreting gas is unavoidable in most cases, there are two possible answers: *i*) these objects are stable (either because deviations from the Kerr metric are not small or because they are not surrounded by an ergoregion) or *ii*) they are indeed unstable. In the latter case, they should decay (probably spin down, as neutron stars do [22]) to a stable configuration emitting gravitational waves. If we consider the large number of expected super-massive objects in our Universe, there might exist a relevant background of low-frequency ($\nu \lesssim 1/M$) cosmic gravitational waves.

The possibility that most of the super-massive objects in galactic nuclei are not BHs, but super-spinning bodies, is quite relevant and cannot be ignored in view of future experiments like the gravitational wave detector LISA or sub-millimeter very long baseline interferometry facilities. In the literature, there are several works discussing how we can test the Kerr nature of these super-massive objects, see e.g. Refs. [23, 24]. For example, by observing the inspiral of a 10 Solar masses object into a 10^6 Solar masses object for one year, LISA will be able to constrain the quadrupole moment of the super-massive body with a precision of 10^{-3} [25]. Surprisingly, all these works always assume that $|a_*| < 1$. On the other hand, the conclusion of this letter is that if we consider the possibility that these objects are not BHs, we cannot restrict our attention to the case $|a_*| < 1$.

Conclusions — BHs in general relativity must satisfy the bound $|a_*| \leq 1$, where a_* is the spin parameter. This is just the condition for the existence of the event horizon. The accretion process can spin a BH up to $a_* \approx 0.998$ and we currently believe that most of the super-massive compact objects in galactic nuclei are rapidly rotating super-massive BHs. In this letter I suggested that deviations from the Kerr metric may have the accretion process spin the compact object up to $a_* > 1$. If the super-massive objects at the center of galaxies are not the black holes predicted by general relativity, their spin parameter could violate the constraint $|a_*| \leq 1$. Unfortunately, we cannot predict the equilibrium spin parameter for an object with a given quadrupole moment, because the MN solution is in quasi-cylindrical coordinates and requires $|a_*| < 1$. An extension to include super-spinning objects is definitively non-trivial, but might not be impossible [26], at least numerically.

The good news of this story is that an anomalous quadrupole moment may induce an anomalously high value of the spin, whose general relativistic effects would be larger and thus easier to discover.

I would like to thank Enrico Barausse and Naoki Yoshida for critically reading a preliminary version of this manuscript and providing useful feedback. This work was supported by World Premier International Research Center Initiative (WPI Initiative), MEXT, Japan, and by the JSPS Grant-in-Aid for Young Scientists (B) No.

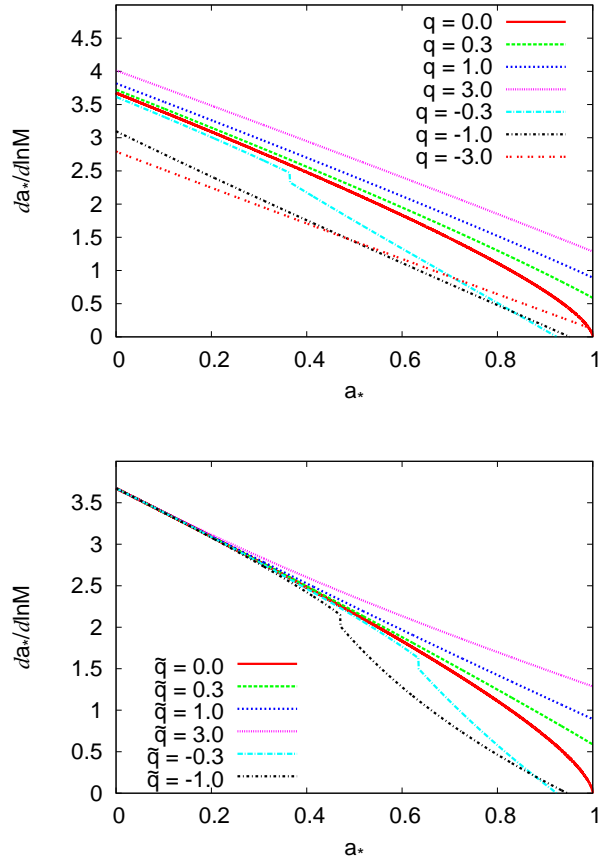


FIG. 1. $da_*/d \ln M$ as a function of the spin parameter a_* for several values of the anomalous quadrupole moment q (upper panel) and \tilde{q} (lower panel). As long as $da_*/d \ln M > 0$, the gas of the disk spins the object up. The fact that $da_*/d \ln M > 0$ at $a_* = 1$ suggests that the equilibrium is reached for $a_*^{eq} > 1$, which is impossible for black holes.

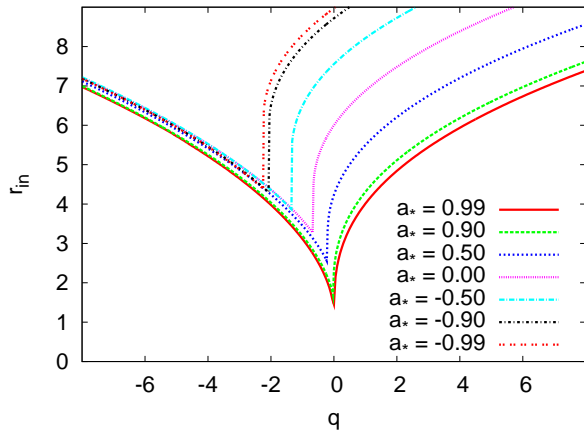


FIG. 2. Inner radius of the disk in Schwarzschild coordinates as a function of the anomalous quadrupole moment q (the case $q = 0$ corresponds to a black hole) for different values of the spin parameter a_* . Inner radius in units $M = 1$.

22740147.

* cosimo.bambi@ipmu.jp

- [1] A. P. Cowley, *Ann. Rev. Astron. Astrophys.* **30**, 287 (1992).
- [2] J. Kormendy and D. Richstone, *Ann. Rev. Astron. Astrophys.* **33**, 581 (1995).
- [3] C. E. . Rhoades and R. Ruffini, *Phys. Rev. Lett.* **32**, 324 (1974); V. Kalogera and G. Baym, *Astrophys. J.* **470**, L61 (1996).
- [4] E. Maoz, *Astrophys. J.* **494**, L181 (1998).
- [5] Y. Sekiguchi and M. Shibata, arXiv:1009.5303 [astro-ph.HE].
- [6] K. Kiuchi, Y. Sekiguchi, M. Shibata and K. Taniguchi, *Phys. Rev. D* **80**, 064037 (2009).
- [7] S. A. Hughes and R. D. Blandford, *Astrophys. J.* **585**, L101 (2003).
- [8] C. F. Gammie, S. L. Shapiro and J. C. McKinney, *Astrophys. J.* **602**, 312 (2004).
- [9] F. Pretorius, in *Physics of Relativistic Objects in Compact Binaries: from Birth to Coalescence*, edited by Colpi et al. (Springer Verlag and Canopus Publishing Limited, Bristol, UK, 2009), pp. 305-369.
- [10] E. Berti and M. Volonteri, *Astrophys. J.* **684**, 822 (2008).
- [11] J. M. Bardeen, *Nature* **226**, 64 (1970).
- [12] K. S. Thorne, *Astrophys. J.* **191**, 507 (1974).
- [13] S. L. Shapiro, *Astrophys. J.* **620**, 59 (2005).
- [14] M. Volonteri, P. Madau, E. Quataert and M. J. Rees, *Astrophys. J.* **620**, 69 (2005).
- [15] C. Bambi and K. Freese, *Phys. Rev. D* **79**, 043002 (2009); C. Bambi, K. Freese, T. Harada, R. Takahashi and N. Yoshida, *Phys. Rev. D* **80**, 104023 (2009); C. Bambi, T. Harada, R. Takahashi and N. Yoshida, *Phys. Rev. D* **81**, 104004 (2010); C. Bambi and N. Yoshida, *Phys. Rev. D* **82**, 064002 (2010); C. Bambi and N. Yoshida, *Phys. Rev. D* **82**, 124037 (2010).
- [16] V. S. Manko and I. D. Novikov, *Class. Quant. Grav.* **9**, 2477 (1992).
- [17] C. Bambi and E. Barausse, *Astrophys. J.* **731**, 121 (2011).
- [18] W. G. Laarakkers and E. Poisson, *Astrophys. J.* **512**, 282 (1999).
- [19] P. A. G. Scheuer and R. Feiler, *Mon. Not. Roy. Astron. Soc.* **282**, 291 (1996).
- [20] P. Natarajan and J. E. Pringle, *Astrophys. J.* **506**, L97 (1998).
- [21] P. Pani, E. Barausse, E. Berti and V. Cardoso, *Phys. Rev. D* **82**, 044009 (2010).
- [22] N. Andersson, D. I. Jones, K. D. Kokkotas and N. Stergioulas, *Astrophys. J.* **534**, L75 (2000).
- [23] N. A. Collins and S. A. Hughes, *Phys. Rev. D* **69**, 124022 (2004); K. Glampedakis and S. Babak, *Class. Quant. Grav.* **23**, 4167 (2006); J. R. Gair, C. Li and I. Mandel, *Phys. Rev. D* **77**, 024035 (2008); T. A. Apostolatos, G. Lukes-Gerakopoulos and G. Contopoulos, *Phys. Rev. Lett.* **103**, 111101 (2009); S. J. Vigeland and S. A. Hughes, *Phys. Rev. D* **81**, 024030 (2010).
- [24] C. Bambi and N. Yoshida, *Class. Quant. Grav.* **27**, 205006 (2010); T. Johannsen and D. Psaltis, *Astrophys. J.* **718**, 446 (2010).

- [25] L. Barack and C. Cutler, Phys. Rev. D **75**, 042003 (2007).
- [26] M. Calvani, R. Catenacci and F. Salmistraro, Lett. Nuovo Cim. **16**, 460 (1976); V. S. Manko and C. Moreno, Mod. Phys. Lett. A **12**, 613 (1997).